

INTEGRATED DETERMINATION OF THERMOPHYSICAL
CHARACTERISTICS OF SOLID AND GRANULAR
MATERIALS AT ROOM TEMPERATURE

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A method is described for the integrated determination of the thermophysical characteristics of solid and granular materials at room temperature.

We know of many methods of determining thermophysical characteristics; these methods assume the use of surface or internal sources of constant heat. In the presence of heat sinks, the final effect resulting from the development of a temperature field in this case will be a steady thermal state, in which regime the thermal conductivity is usually determined (the plate and hollow-cylinder method) [1]. The drawback of this approach lies in the fact that it does not make use of the nonsteady phase of the thermal process, which basically enables us to determine the thermal diffusivity. A great variety of procedures are possible for calculating the thermal conductivity and diffusivity.

It is the purpose of the present article to describe an integrated method and device for the determination of the thermophysical characteristics of solid and granular materials in the presence, respectively, of time-constant heat sources and sinks within the test specimen and at its surface. The method is based on the familiar solutions of the heat-conduction equation for unbounded plates and a hollow cylinder, when these bodies are acted on by a constant-power source, and when the outside surfaces are kept at a constant temperature [2]. Analysis of these and similar solutions demonstrates that beginning with some instant of time, at any point on the above-indicated materials, the following [3, 4] equations will be valid:

$$m = \frac{\ln(t_{st} - t_1) - \ln(t_{st} - t_2)}{\tau_2 - \tau_1} = \frac{\ln b_1 - \ln b_2}{\tau_2 - \tau_1} \quad (1)$$

In expression (1) t_{st} is the temperature difference in the steady-state, and it is a function of the Kirpichev number; t_1 and t_2 are the values of the temperature difference for the time τ_1 and τ_2 , respectively; b is the rate of temperature change. Since reference [5] described a method for the integrated determination of the thermophysical characteristics with the use of (1) for plane-parallel specimens, we will dwell on the analysis of the solution for a hollow cylinder, which we used in the method for the investigation of the thermophysical properties of granular materials.

Let us examine a hollow unbounded cylinder whose inside radius is r_0 and whose outside radius is R . The inside surface of the cylinder is acted on by a constant heat flow q and the outside surface is kept at a constant zero temperature. The initial temperature is equal to zero. The solution of the stated problem is well known [2] and can be presented in the form

$$t(r, \tau) = t_{st} \left\{ 1 + \frac{\pi R}{r_0 \ln \frac{R}{r}} \sum_{n=1}^{\infty} \frac{J_0^2(x_n) \left[J_0\left(\frac{r}{R} x_n\right) Y_1(kx_n) - J_1(kx_n) Y_0\left(\frac{r}{R} x_n\right) \right]}{x_n \left[J_1^2(kx_n) - J_0^2(x_n) \right]} \exp(-x_n^2 Fo) \right\} \quad (2)$$

where

$$t_{st} = \frac{r_0 q}{\lambda} \ln \frac{R}{r}; \quad Fo = \frac{\alpha \tau}{R^2}; \quad k = \frac{r_0}{R},$$

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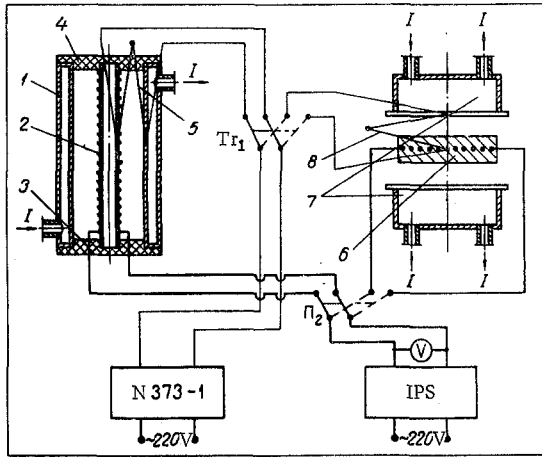


Fig. 1

Fig. 1. Diagram of the experimental installation for integrated research into thermophysical characteristics, at room temperature: 1) cylindrical container; 2) low-inertia heater; 3, 4) covers; 5, 8) differential thermocouples; 6) flat specimen with imbedded heater; 7) cooling units; I) cooling water.

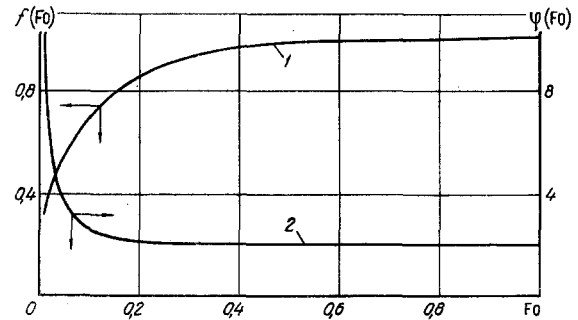


Fig. 2

Fig. 2. Function (8) for the point $r = r_0$ (curve 1) and function (9) for the point $r = R/2$ (curve 2).

where x_n denotes the roots of the characteristic equation

$$J_0(x) Y_1(kx) - J_1(kx) Y_0(x) = 0. \quad (3)$$

In the experimental installation which we employed, the ratio of the inside radius for the hollow cylinder to the outside radius is equal to one fourth ($k = r_0/R = 0.25$). For this ratio the value of the first root in (3) is $x_1 = 2.668$.

In order to introduce the heating rates in the form of (1), we have to evaluate the convergence of series (2). The calculations carried out for $k = 0.25$ show that when $Fo \geq 0.05$ for a point $r = r_0$ with accuracy to 0.5% we can limit ourselves to the first nonsteady number of series (2). Consequently,

$$t(r, \tau) - t_{st} = A_1 \exp(-x_1^2 Fo). \quad (4)$$

We can easily find the formula for the determination of the thermal diffusivity from (4):

$$a = \frac{R^2}{x_1^2} m. \quad (5)$$

Regardless of the method used to determine m (see (1)) by experimentation, we must know the nature of the change in the temperature difference between any two points of the specimen.

The experimental scheme by which it is possible to achieve integrated determination of the thermophysical characteristics of solid and granular materials is shown in Fig. 1. In studying solid materials we place specimen 6 together with the heater into the space between "cooling units" 7 through which we pass water I at a constant temperature, and this water is compressed by the sides of the cooling unit. Since the experimental method has been described earlier in [5], we will dwell in greater detail on the description of the "calorimeter" for the testing of the granular materials.

The granular materials are studied in cylinder 1 whose outside surface is flushed by water I at constant temperature. Low-inertia heater 2, made of a manganin wire 0.2 mm in diameter, is positioned on the axis of the cylinder. A thick-walled copper tube serves as the frame of the heater.

The heater is held in place by means of bottom and top covers 3 and 4. The junctions of the differential Chromel-Alumel thermocouple 5 are mounted at the heater surface and on the inside surface of the cylinder, i.e., at the point $r = r_0$ and $r = R$.

The experiment was carried out in the following sequence. Having removed the upper cover, we filled the cavity between the outside surface of the heater and the inside surface of the cylinder with the material being tested. The surface of the cylinder was cooled with running water to level off the temperature throughout the volume of the material. A constant-power source (heater 2) was then switched on. The

change in the temperature difference is recorded on the graph-paper strip of an automatic N 373-1 photoelectric recorder whose lower voltage limit is 0.5 mV. The steady-state regime sets in at $Fo \geq 1.0$. Knowing the temperature at any instant of time and in the steady state, we can plot the function $\ln(t_{st} - t) = f(\tau)$. In graphic form, this function is a straight line whose slope is given by the thermal diffusivity (see [1, 5]). The thermal diffusivity can be calculated directly from formula (5), eliminating the need for the plotting of the curve. The reckoning in this case must proceed from $t \geq t_{st}/2$, which corresponds to $Fo \geq 0.05$ (the regular regime).

Knowing the heat flow and the temperature difference in the steady state, we can find the thermal conductivity

$$\lambda = \frac{r_0 q}{\Delta t_{st}} \ln \frac{R}{r_0}. \quad (6)$$

The specific heat capacity is found from the familiar relationship

$$c = \frac{\lambda}{a\gamma}. \quad (7)$$

Let us dwell briefly on other methods of calculating the thermophysical properties. It follows from relationship (2) that

$$\frac{t(r, \tau)}{t_{st}(r, \infty)} = f(Fo), \quad (8)$$

where

$$t_{st}(r, \infty) = \frac{r_0 q}{\lambda} \ln \frac{R}{r}.$$

Function (8), calculated for the point $r = r_0$, is shown in Fig. 2 (curve 1). Having experimentally determined the ratio in (8), we can find the Fo number and, consequently, we can find the thermal diffusivity. The thermal conductivity and heat capacity in this case are also determined from (6) and (7). An analogous method of calculation was employed in studying materials in the form of plates [6].

Solution (2) makes it possible to achieve purely nonsteady variants of investigation. One of these involves the determination of the function $\ln b = f(\tau)$ (a straight line). However, the determination of $\ln b$ involves substantial errors.

The essence of the second variant involves the following. From (2) we have the ratio of temperatures at any two points at the identical instants of time, again a function of the Fo number, i.e.,

$$\frac{t(r_1, \tau)}{t(r_2, \tau)} = \varphi(Fo). \quad (9)$$

In Fig. 2 (curve 2) function (9) has been plotted for the point $r = r_0$ and $r = R/2$. If we know the experimental value of (9) and use the curve of (2) we can find the Fo number, and then we find the thermal diffusivity. Since the value of $f(Fo)$ for the point $r = r_0$ is known (curve 1), the thermal conductivity can be found directly from (2).

The shortening of all these calculation methods, including the one which we have proposed, lies in the fact that the tabular or graph material is derived for fixed points on the assumption of a uniform initial temperature distribution. This latter circumstance requires that these conditions be maintained during the experiment.

The methods and installation for the integrated study of the thermophysical characteristics of solids and granular materials at room temperature, as described here, are unique in their simplicity and reliability. Before examining the errors in the experiment, we will offer certain data. In our installation $r_0 = 5.7$ mm, $h = 138$ mm (heater height), $R = 22.8$ mm, the temperature of the running water is 12°C , and the temperature difference recorded by the differential thermocouple is $8-12^\circ\text{C}$. The "cooling units" in which the solid materials are tested are designed to use plane-parallel circular or square plates up to 70 mm in diameter. Since all of the theoretical relationships are derived from one-dimensional solutions, the selection of the optimum relationships between the linear specimen dimensions is based on the solution and analysis of corresponding two-dimensional problems (see, for example, [7]). For flat specimens the ratio of the

overall height to the diameter of the specimen must be equal to or less than one third. For a cylinder this ratio must be on the order of 3. With such relationships and given the transfer of heat from the ends of the specimens to the ambient medium under conditions of free convection at room temperature the one-dimensionality in the center portions of the specimen housing the sensors remains virtually inviolate. (This is confirmed as well by the work done by other authors [8].) These conclusions are valid for slight temperature differences, which was precisely what we found in our experiments (the heater actually "functioned" at room temperature). As regards the thermal resistances, it is extremely difficult to speak of their effect when studying granular material. In testing solids we implemented measures to reduce the thermal resistance by carefully treating the rubbing surfaces with lubricants and by compressing the specimen. The experiment showed that in testing heat insulators ($\lambda < 1 \text{ W/m} \cdot \text{deg}$) the effect of the thermal resistance on the accuracy with which the thermophysical characteristics are determined under these conditions is insignificant (less than 1%). Thus it is assumed that there is no error resulting from the fact that there is a difference between the actual conditions of the experiment and those theoretically postulated (the unboundedness of the specimen, ideal thermal contact). As regards the relative instrumentation error, its value is a function of the accuracy with which we have measured the quantities contained in the theoretical formulas. In the experiments which we carried out for a large variety of granular and solid materials (quartz sand, common salt, citric acid, polymethyl methacrylate, polytetrafluoroethylene, etc.) the instrumentation error was on the order of 5-7%, while the divergence from data of other authors did not exceed 10%.

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